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The determination of the demagnetization factor resulting from shape anisotropy in ferrite magnets

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The energy equation which yields the expression, $H_{ci} = 0.48[2K_1/M_s - 4\pi M_s]$, for the intrinsic coercive force of $BaFe_{12}O_{19}$ has been used as the basis for numerous investigations. There is reason to believe, however, that the term, 4π, in this expression should be replaced by 2π. To settle this question, the energy equation is resolved to obtain expressions for both the easy and hard directions of magnetization. The resultant equations are used to establish a procedure for the determination of the demagnetization factor resulting from shape anisotropy in ferrite magnets.

A related method is introduced to calculate the saturation magnetization of ferrite magnets. It is demonstrated that M_s values derived by means of this procedure deviate from single-crystal values by $\pm$ 4 parts per thousand, maximum. This method uses magnetic measurements made in the field, $H_a = H^A/2$ for both the hard and easy directions of magnetization.

The evidence obtained leads to the conclusion that, $H_{ci} = 0.48[2K_1/M_s - 2\pi M_{sc}]$, for the Stoner-Wohlfarth spherical model.

1. Introduction

$BaFe_{12}O_{19}$, $PbFe_{12}O_{19}$, and $SrFe_{12}O_{19}$ magnets are comprised of platelet-shaped crystallites whose c-axes are assumed [1] to be normal to the planar surfaces. It is also assumed that the crystal and shape anisotropies of the platelet-shaped particles oppose so their effective anisotropy field, $H_{eff}{}^A$, is given by the expression [1],

$$H_{eff}{}^A = H^A - DM_s . \quad (1)$$

In this equation, the anisotropy field, $H^A = 2K_1/M_s$ [2], and D is the demagnetization factor resulting from shape anisotropy. The term, K_1, introduced here is the first-order anisotropy constant, and M_s is the spontaneous magnetization.

Since $D_a = 4\pi$ for flat plates infinite in extent [3], Mee and Jeschke [4] assumed that $D \approx 4\pi$ for a cylindrical specimen comprised of unoriented, thin platelet-shaped ferrite crystallites. It is evident that this is a reasonable assumption to make because the lead these authors established is still followed [5]. It should be noted, however, that the measured coercivity values for $SrFe_{12}O_{19}$ displayed in Fig. 2 of their report [4] are greater than the corresponding theoretical values. These theoretical values derived on the assumption that $D = 4\pi$ are consequently open to question. It seems more likely that the value, $D = 2\pi$, should be used when Equation 1 is applied to polycrystalline ferrite specimens. The reason for this is discussed in this report.

It will be recognized that resolving this question has considerable theoretical significance. Thus, the intrinsic coercive force given [4] by the expression, $H_{ci} = 0.48\ [H^A - 4\pi M_s]$, is seriously affected by replacing 4π with 2π. It is also important to resolve this question from an applied magnetics point of view since the equation used [6] to compute the saturation magnetization of ferrite magnets is based on relationships between D, N and $\bar{M}$. Here, N is the ballistic demagnetization factor of the magnet, and $\bar{M} = (M_e + M_h)/2$ is its average magnetization in the applied field, $H_a = H^A/2$. In this expression, M_e is the magnetization measured in the easy direction of magnetization, and M_h is the analogous term for the hard direction.

This report is concerned with resolving the

question raised and supporting the answer obtained with experimental and theoretical evidence.

2. The demagnetization factor, *D*, of ferrite magnets

In order to establish the value of D for ferrite magnets, the energy equation,

$$E = K_1 \sin^2 \theta + K_2 \sin^4 \theta - HM_S \cos(\alpha - \theta), \quad (2)$$

for magnetization by the rotational process, is differentiated with respect to θ and the product equated to zero. In this expression which considers only the energy of magnetic interaction and the crystal anisotropy energy, K_1 is the first-order and K_2 is the second-order anisotropy constant. The c-axis of the uniaxial single-crystal to which Equation 2 applies is oriented α radians with respect to the field, H, and θ is the angle between its magnetization vector, $\mathbf{M}_S$, and its c-axis.

When $K_1 \gg K_2$, which is the case for $BaFe_{12}O_{19}$ and $SrFe_{12}O_{19}$ single-crystals [7], the relationship,

$$\left(\frac{\partial E}{\partial \theta}\right)_\alpha = H^A \sin 2\theta - 2H \sin(\alpha - \theta) = 0, \quad (3)$$

is used to establish the value of D.

Fig. 1 is now introduced to show the magnetization vector, $4\pi\mathbf{M}_S$, and the vector fields of an arbitrarily selected crystallite within the magnet. All vectors shown lie in the plane defined by the c-axis of the crystallite and its magnetization vector, $\mathbf{M}_S$. In this figure, however, $4\pi\mathbf{M}_S$ is shown instead of $\mathbf{M}_S$ to permit all vectors with the exception of $\mathbf{H}^A$, to be drawn to scale.

It is seen that the field, $\mathbf{H}$, in the interior of the magnet is the vector sum of the applied field, $\mathbf{H}_a$, and the demagnetizing field, $N\mathbf{M}_S$ [8]. This demagnetizing field, however, has a component, $\mathbf{H}_D = -N\mathbf{M}$, in the field direction [3], so that the actual field in the crystallite acting in the field direction is $H_a - NM$ [8]. It will be recognized that the familiar equation, $H_e = H_a - (B - H_e)N/4\pi$, which is used to express the ballistic demagnetization factor for ellipsoids and cylinders [9], yields the same result. In this expression, H_e is the effective field in the centre of the specimen, and B is its magnetic induction.

It is readily proved that the difference between H and $H_e - NM$ is so small it can be ignored in many practical applications. In others, compensation can be made by considering the difference in the directions of $\mathbf{H}$ and $\mathbf{H}_a$. For the case illustrated by Fig. 1, $H = 0.968\,(H_a - NM)$. This relationship was derived on the assumption that the specimen is a sphere and that $H_a = H^A/2$. This difference becomes less as α decrease, Thus H approach $H_a - NM$ as its limiting value as α approaches zero.

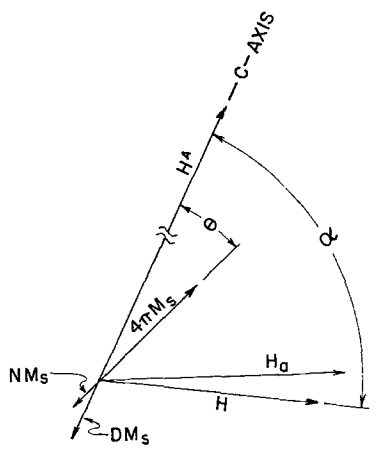

Figure 1 The magnetization vector and the vector fields of a crystallite in the interior of a $BaFe_{12}O_{19}$ magnet.

Therefore, to establish the value of D by means of Equation 3 to a good first approximation, H is replaced by $H_a - NM$ and $H_{eff}^A = H^A - DM_S$ is substituted for H^A. The resultant expression follows:

$$(H^A - DM_S) \sin 2\theta = 2(H_a - NM) \sin(\alpha - \theta). \quad (4)$$

This equation applies to a thin platelet-shaped crystallite whose $\mathbf{DM}_S$ vector coincides with its c-axis. Microphotographs [10] show, however, that the crystallites comprising ferrite magnets do not have this shape. Consequently, the demagnetization factor resulting from shape anisotropy must have a component which lies in the equatorial plane as well as the one which coincides with the c-axis.

The magnitude of these components is determined by applying orthogonal fields to the magnet to obtain the data needed to solve the following independent expressions simultaneously:

$$(H^A - D_a M_S) \sin 2\theta = 2(H_a - NM_e) \sin(\alpha - \theta), \quad (4a)$$

and

$$(H^A - D_b M_s) \sin 2\phi = 2(H^A - NM_h) \sin(\psi - \phi) . \quad (4b)$$

In these equations, D_a is the demagnetization factor resulting from shape anisotropy for the easy direction, and D_b is the analogous term for the hard direction of magnetization [11]. The corresponding magnetization terms are, M_e and M_h, respectively [6]. The angles, ψ and ϕ are the orthogonal counterparts of α and θ.

It is self-evident that, if these equations are valid, they must hold for both isotropic and oriented magnets of all shapes, placed in fields of arbitrary value. Consequently, they must hold when

$$(H_a - NM_e)/(H^A - D_a M_s) = 1/2 , \quad (5a)$$

and

$$(H_a - NM_h)/H^A - D_b M_s) = 1/2 . \quad (5b)$$

These conditions are imposed on Equation 4a and b, thereby making $\alpha = 3\theta$ and $\psi = 3\phi$ so that these equations are completely satisfied. This result permits Equation 5a and b to be solved simultaneously to obtain an expression for D_a and D_b free of trigonometric relationships. When this is done and H_a is equated to $H^A/2$, it is found that in the field, $H_a = H^A/2$,

$$\bar{M} = (M_e + M_h)/2 = \bar{D} M_s/2N . \quad (6)$$

In this expression, $\bar{D} = (D_a + D_b)/2$.

In order to see if this equation yields reasonable values, it is assumed that an isotropic spherical specimen is placed in the field, $H_a = H^A/2$, after it has been magnetically saturated. Under these conditions, $\bar{M}/M_s = 0.761$ for the Stoner-Wohlfarth model [12]. This compares well with, $\bar{M}/M_s = 3/4$ obtained by substituting the terms, $N = 4\pi/3$ for a sphere [3], and $\bar{D} = 2\pi$ into Equation 6. $\bar{D} = 2\pi$ because $D_a \approx 4\pi$ for thin platelet-shaped ferrite particles [4], consequently $D_b \approx 0$.

It should be noted that the demagnetization factor, $\bar{D} = 2\pi$, corresponds to the one derived by means of the free energy equation for demagnetization, $E_D = M_s^2 VD/2$. For convenience, this derivation is given in Appendix I.

3. Experimental

The experimental data to be used in support of Equation 6 were obtained on the basis of the well-established principle that the saturation magnetization of a specimen is a vector quantity. As a consequence, its magnitude cannot be established from low-field measurements made in only one direction unless the specimen evaluated is both spherical and isotropic.

Isotropic ferrite magnets, however, are difficult, if not impossible, to produce. This difficulty occurs because these magnets are usually made from calcines which are readily oriented by mechanical means [13]. As a consequence, the specimen produced displays two hysteresis loops, one for the easy direction of magnetization and another one for the hard. Data derived from loops of both types are used in support of Equation 6.

Fig. 2 displays hysteresis loops of a $SrFe_{12}O_{19}$ magnet made in the laboratory using procedures intended to minimize orientation. The upper hysteresis loop shown is for the easy direction of magnetization. This is observed when the field is applied to the specimen in the same direction used in compacting the calcine from which it was made. The lower loop, however, is obtained by applying the field normal to this direction.

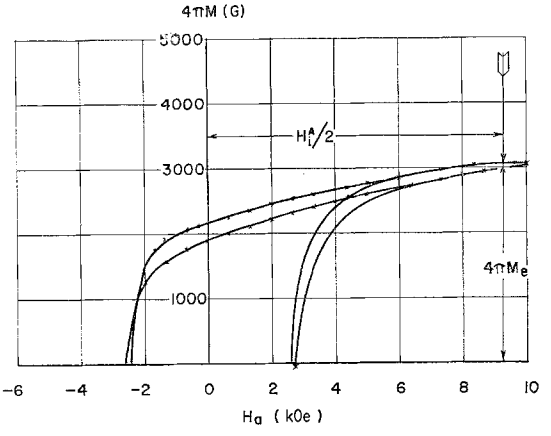

Figure 2 Hysteresis loops of a $SrFe_{12}O_{19}$ magnet specially prepared to minimize its orientation.

It will be noticed that the hysteresis loop for the easy direction becomes essentially parallel to the H_a-axis at the field value indicated by the arrow. This occurs [6] when $H_a/H_i^A = H_a/H^A = 1/2$. The field, H_i^A, is consequently referred to as the "indicated" anisotropy field.

The hysteresis loops shown in Fig. 2, as well as those displayed in Fig. 3, were obtained by means of a Magnemetrics Hysteresisgraph with pole coils, using procedures which have been described [6] and should become apparent by referring to these figures. The magnets used to

TABLE I The characteristics of the test magnets

Type magnet	Density (g cm^{-3})	$4\pi M_e$ (G)	$4\pi M_h$ (G)	$4\pi M_{re}$ (G)	$4\pi M_{rh}$ (G)	$(B_dH_d)_m$ (MG–Oe) (easy)	H_{ci} (Oe) (easy)
$BaFe_{12}O_{19}$[I]	5.18	4070	2470	3780	640	3.50	2300
$BaFe_{12}O_{19}$[III]	4.91	3200	3010	2200	1840	0.96	1950
$BaFe_{12}O_{19}$[IV]	5.00	3215	3085	2240	—	0.93	1610
$SrFe_{12}O_{19}$[II]	4.67	3080	2985	2160	1900	1.05	2450
$SrFe_{12}O_{19}$[V]	4.77	3160	3040	2360	1880	1.15	1730
$SrFe_{12}O_{19}$[VI]	4.87	3290	3080	2380	1840	1.20	2180
$SrFe_{12}O_{19}$[VII]	4.90	3295	3065	1200	760	—	300

[I]Oriented magnet; contains 2.3 wt% impurities.
[II]Sintered @ 1210°C–2 h; contains 2.8 wt% impurities.
[III]Sintered @ 1250°C–2 h; contains 2.8 wt% impurities.
[IV]Sintered @ 1250°C–48 h; contains 2.8 wt% impurities.
[V]Sintered @ 1285°C–2 h; contains 2.8 wt% impurities.
[VI]Sintered @ 1315°C–2 h; contains 2.8 wt% impurities.
[VII]Sintered @ 1315°C–48 h; contains 2.5 wt% impurities.

obtain these loops, as well as the others whose characteristics are reported in Table I, were approximately cubic in shape and unground.

The hysteresis loops displayed in Fig. 2 are those of a $SrFe_{12}O_{19}$ magnet fired to a peak temperature of 1285°C for 2 h. The hysteresis loops shown in Fig. 3, however, are those of a $SrFe_{12}O_{19}$ magnet held at the peak temperature, 1315°C, for 48 h to cause marked crystal growth and consequent extreme shearing of its hysteresis loops. In spite of this severe treatment, its hysteresis loop for the easy direction followed the normal pattern [6] and became essentially parallel to the H_a-axis at the field value indicated by the arrow.

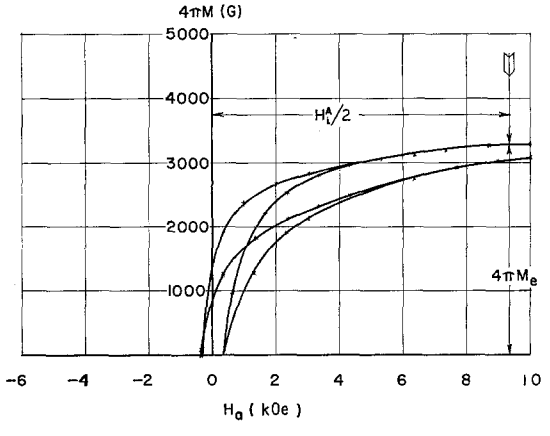

Figure 3 Hysteresis loops of a $SrFe_{12}O_{19}$ magnet grossly overfired to cause extreme shearing of its magnetization curves.

These figures show graphically how the $4\pi M_e$ values reported in Table I were measured at $H_a = H_i^A/2$. In an analogous manner, the tabulated $4\pi M_h$ quantities were measured at this field value.

Included in Table I are data which are used to normalize the intrinsic induction values of the magnets with respect to their X-ray densities and ferrite contents. The density values listed were obtained by displacement methods and are believed to be accurate to ± 0.1%. The ferrite content reported for the magnet was determined by finding the impurities present by chemical and spectrographic analyses and subtracting the sum from 100%. The results reported are considered accurate to ± 0.1%.

4. Basic considerations

Equation 6 has been shown to give reasonable results only for the case where the theoretical specimen is spherical and isotropic. It is obvious, therefore, that before the experimental values, $4\pi M_e$ and $4\pi M_h$ given in Table I, can be used in support of Equation 6 with confidence, it must be proved that $\bar{M} = (M_e + M_h)/2$ is constant for theoretical oriented magnets placed in the field, $H_a = H^A/2$. Then to have significance, this constant must be evaluated for cubic magnets.

To meet these objectives, it is assumed that the hysteresis loops of an oriented spherical specimen correspond to those of ellipsoids comprised of randomly oriented crystallites. Thus, the hysteresis loop of a selected prolate ellipsoid coincides with that observed for the sphere's easy

direction when the field is applied parallel to the major axis of the ellipsoid. However, the hysteresis loop observed when the applied field is parallel to the minor axis of a selected oblate ellipsoid duplicates that for the sphere's hard direction. The ellipsoids referred to are generated by rotating a selected ellipse about its major and minor axis, respectively.

It is proved that $\bar{M} = (M_e + M_h)/2$ equals a constant for ellipsoid pairs by computing their $\bar{M}_e/M_s$ and $\bar{M}_h/M_s$ values in the field, $H = H^A/2$, and averaging the results. Here, the terms, $\bar{M}_e$ and $\bar{M}_h$, are the weighted, averaged magnetization values in the field direction. The results obtained indicate that $\bar{M} = (M_e + M_h)/2$ is constant for theoretical, oriented spherical specimens because the eccentricity of the ellipse used to generate the ellipsoids considered varies from 0 to 0.996.

Analogous reasoning is used to prove that $\bar{M} = (M_e + M_h)/2$ is constant for cylindrical specimens. In this case, however, it is assumed that the hysteresis loops of oriented cubic magnets coincide with those of cylinders comprised of randomly oriented crystallites.

5. Magnetization equations

In order to derive the equations needed to provide the promised proof, it is assumed that a cylindrical specimen is comprised of uniaxial crystallites whose c-axes are randomly oriented. An incremental volume,

$$\Delta V = \rho^2 \sin\alpha \Delta\rho \Delta\alpha \Delta\phi\,, \tag{7}$$

in the specimen contains its portion of these crystallites since the specimen is assumed to be homogeneous. In this expression, ρ is the distance from the origin to the incremental volume, and α is the angle between the radius vector ρ and the specimen's axis. This axis is oriented so that it is parallel to the applied field, H_a. The angle, $\Delta\phi$, refers to the rotation of ρ about the specimen's axis.

Following the lead of Langevin [14] and Stoner and Wohlfarth [12], it is assumed that magnetic interactions between the crystallites can be neglected and that statistical means can be used to obtain the weighted average magnetization of the specimen. Thus, it is assumed that the probable number of crystallites in the differential volume, $dV = (2\pi/3)\rho^3 \sin\alpha\, d\alpha$, whose c-axes are oriented α radians with respect to the field direction, is proportional to the volume subtended by the angle, $d\alpha$. The product of this number and the magnetization, $M = M_s \cos(\alpha - \theta)$, of each of these crystallites in the field direction is summed over all values of α. The resultant sum when divided by the volume of the specimen yields the weighted average magnetization in the field direction.

In order to illustrate this procedure which is new, the parameters defined by Equation 8 are used. Thus, the distance, r, between the axis of revolution of the cylindrical specimen and the incremental volume, ΔV, is given as, $r = \rho \sin\alpha$. The perpendicular distance, z, between this incremental volume and the basal plane which bisects the cylinder is expressed as, $z = \rho \cos\alpha$.

The ellipsoids considered are generated by rotating an ellipse about its major and minor axes, respectively. The polar equation used when the ellipse is rotated about its major axis is $\rho^2 = b^2(1 - e^2)/(1 - e^2\cos^2\alpha)$, and $\rho^2 = a^2(1 - e^2)/(1 - e^2\sin^2\alpha)$, when its minor axis serves as the axis of rotation. In these expressions, e is the eccentricity of the ellipse whose major and minor axes are, respectively, b and a.

The values of ρ from these polar equations are used in the expression, $dV = (2\pi/3)\rho^3 \sin\alpha\, d\alpha$, and the relationship, $M = M_s \cos(\alpha - \theta)$, is invoked to derive equations which express the magnetization of ellipsoids of revolution. These equations, given below, are based on the assumption that the applied field is parallel to the axis of rotation

$$\bar{M}_e/M_s \int_0^{\pi/2} \frac{\sin\alpha\, d\alpha}{[1 - e^2\cos^2\alpha]^{3/2}} = \int_0^{\pi/2} \frac{\cos(\alpha - \theta)\sin\alpha\, d\alpha}{[1 - e^2\cos^2\alpha]^{3/2}}\,, \tag{8}$$

and

$$\bar{M}_h/M_s \int_0^{\pi/2} \frac{\sin\alpha\, d\alpha}{[1 - e^2\sin^2\alpha]^{3/2}} = \int_0^{\pi/2} \frac{\cos(\alpha - \theta)\sin\alpha\, d\alpha}{[1 - e^2\sin^2\alpha]^{3/2}}\,. \tag{9}$$

The term, $\bar{M}_e$, is the weighted average magnetization for the specimen in its easy direction of magnetization, and $\bar{M}_h$ is the analogous term for the hard direction. These equations, obviously, apply only to specimens that have been magnetically saturated.

When the demagnetization curves derived by means of these equations superimpose, the specimens they represent are isotropic. However, $\bar{M}_e$ can only be made to equal $\bar{M}_h$ by making

$e = 0$. Consequently, as e approaches 0 as a limiting value, Equations 8 and 9 yield the Stoner-Wohlfarth equation [12] in the following form:

$$\bar{M}_{th}/M_s = (\bar{M}_e + \bar{M}_h)/2M_s = \int_0^{\pi/2} \cos(\alpha - \theta)\sin\alpha \, d\alpha \cdot \quad (10)$$

The term, $\bar{M}_{th}$ is used here to distinguish the averaged magnetization of theoretical specimens from the term, $\bar{M}$, used in Equation 6.

Procedures analogous to those followed to obtain Equations 8 and 9 are used to derive equations to express the magnetization of cylindrical specimens. In this case, however, $z^3/\cos^3\alpha$ and $r^3/\sin^3\alpha$ are used in sequence to replace ρ^3 in the expression, $dV = (2\pi/3)\rho^3 \sin\alpha \, d\alpha$. The equations derived follow:

$$r_e^2 z_e \bar{M}_e/M_s = 2z_e^3/3 \int_0^{\beta} \frac{\cos(\alpha - \theta)\sin\alpha \, d\alpha}{\cos^3\alpha} + 2r_e^3/3 \int_{\beta}^{\pi/2} \frac{\cos(\alpha - \theta)d\alpha}{\sin^2\alpha}, \quad (11)$$

and

$$r_h^2 z_h \bar{M}_h/M_s = 2z_h^3/3 \int_0^{\pi 2-\beta} \frac{\cos(\alpha - \theta)\sin\alpha \, d\alpha}{\cos^3\alpha} + 2r_h^3/3 \int_{\pi/(2-\beta)}^{\pi/2} \frac{\cos(\alpha - \theta)\, d\alpha}{\sin^2\alpha} \cdot \quad (12)$$

The term, r_e in Equation 11, is the radius of the theoretical cylinder whose weighted average magnetization in the field direction is $\bar{M}_e$. Its length is $2z_e$ and $z_e/r_e = \cot\beta \geqslant 1$. The analogous dimensions of the cylinder whose magnetization is $\bar{M}_h$ are r_h and $2z_h$, respectively. Moreover, $z_h/r_h = \tan\beta \geqslant 1$.

Here, too, when $\bar{M}_e = \bar{M}_h$, the hysteresis loops derived by means of these equations correspond to those of an isotropic magnet. Hence, in the limiting case when $\cot\beta = \tan\beta$,

$$\bar{M}_{th}/M_s = (\bar{M}_e + \bar{M}_h)/2M_s = 2/3 \int_0^{\pi/4} \frac{\cos(\alpha - \theta)\sin\alpha \, d\alpha}{\cos^3\alpha} + 2/3 \int_{\pi/4}^{\pi/2} \frac{\cos(\alpha - \theta)\, d\alpha}{\sin^2\alpha} \cdot \quad (13)$$

6. Results and discussion

Data are presented in Table II to show that $\bar{M}_{th} = (\bar{M}_e + \bar{M}_h)/2$ is constant to within 2% for both cylindrical and ellipsoidal specimen pairs. The $\bar{M}_e/M_s$ and $\bar{M}_h/M_s$ values tabulated were computed by means of Equations 3 and 9 to 13, inclusive for specimens placed in the field, $H = H^A/2$, after they had been magnetically saturated. Since demagnetization factors were not used in the computations, the 1.8% maximum deviation from constancy is ignored and the results taken to indicate that $(\bar{M}_e/\bar{M}_h)/2$ is constant for theoretical cylindrical and ellipsoidal specimens.

Data from Table I are then used to establish the value of this constant for cubic magnets. Thus, the experimental values of $4\pi M_e$ and $4\pi M_h$ for oriented and non-oriented magnets made under different conditions, were normalized with respect to their X-ray densities and ferrite contents. The resultant terms listed in Table III were used to obtain the tabulated values of $\overline{D}/2N$ by means of Equation 6. In these computations, the single-crystal value of $4\pi M_s$ for $BaFe_{12}O_{19}$ and $SrFe_{12}O_{19}$ [7, 15] was used.

It is seen that the maximum deviation of these $\overline{D}/2N$ values from their mean average is less than 4 parts per thousand. This is assumed to prove that, $\bar{M} = (M_e + M_h)/2$ is equal to a constant,

TABLE II The averaged magnetization values, $\bar{M}_{th} = (\bar{M}_e + \bar{M}_h)/2$, of theoretical cylindrical and ellipsoidal specimens placed in the field, $H/H^A = 1/2$, after magnetic saturation

Cylindrical specimens				Ellipsoidal specimens			
$2z_e/r_e$	$\bar{M}_e/M_s$	$\bar{M}_h/M_s$	$(\bar{M}_e+\bar{M}_h)/2M_s$	e^2	$\bar{M}_e/M_s$	$\bar{M}_h/M_s$	$(\bar{M}_h+\bar{M}_h)/2M_s$
1.00	0.788	0.788	0.788	0	0.761	0.761	0.761
1.73	0.856	0.713	0.785	0.89	0.882	0.634	0.758
2.30	0.886	0.677	0.781	0.9845	0.948	0.561	0.754
2.75	0.902	0.656	0.779	0.9922	0.962	0.545	0.753
3.73	0.925	0.624	0.774				

TABLE III The $D/2N$ factors of the test magnets calculated from their $4\pi M_e$ and $4\pi M_h$ values at $H_a/H_i{}^A = 1/2$.

Type magnet	X-ray density (g cm^{-3})	$4\pi M_e$ (normalized)	$4\pi M_h$ (normalized)	$D_a/2N$	$D_b/2N$	$\overline{D}/2N$
$BaFe_{12}O_{19}{}^{I}$	5.28	4245	2575	0.889	0.539	0.714
$BaFe_{12}O_{19}{}^{III}$	5.28	3540	3330	0.741	0.697	0.719
$BaFe_{12}O_{19}{}^{IV}$	5.28	3495	3350	0.732	0.702	0.717
$SrFe_{12}O_{19}{}^{II}$	5.11	3465	3360	0.726	0.704	0.715
$SrFe_{12}O_{19}{}^{V}$	5.11	3485	3350	0.730	0.702	0.716
$SrFe_{12}O_{19}{}^{VI}$	5.11	3550	3325	0.743	0.696	0.719
$SrFe_{12}O_{19}{}^{VII}$	5.11	3525	3280	0.738	0.687	0.713
					Average $\overline{D}/2N = 9/4\pi = 0.716$	

Note: Superscripts refer to footnotes following Table I.

and this constant is $9/4\pi$ for cubic ferrite magnets. Thus,

$$\overline{M} = (M_e + M_h)/2 = 9fM_s/4\pi . \quad (14)$$

In this expression, f is a factor which indicates that $\overline{M}_s$ derived from experimental M_e and M_h values is to be normalized with respect to the magnet's X-ray density and its ferrite content.

This equation is preferred over the one used [6] to compute the unknown M_s values of ferrite magnets with the following empirical chemical formulae: BaO.5.5 Fe_2O_3; $BaFe_{II}U$ = 4 BaO.19Fe_2O_3 [16], and $BaFe_{IV}B$ = 3 BaO.16 Fe_2O_3 [6]. The correction factor for orientation effects included in the earlier equation [6] does not appear in Equation 14. This is because its use cannot be justified on theoretical grounds.

To illustrate the use of Equation 14, it is used to compute the $4\pi M_s$ values of the magnets whose properties are given in Table I. The computed values are compared with the corresponding single-crystal terms [7, 15] in Table IV. It is seen that agreement between the single-crystal values and those computed is extremely good falling within $\pm$ 4 parts per thousand as reported in Table III. Good agreement between the "indicated" anisotropy field, $H_i{}^A$ values and the single-crystal terms [7, 15] is also evident. This confirms earlier observations [6]. All $H_i{}^A$ values reported were derived from hysteresis loops for the easy direction as indicated by Figs. 2 and 3.

7. Conclusion

The evidence presented indicates that Equation 6 is fundamentally sound. It may be used, therefore, to compute the $\overline{D}$ values of magnets when their M_s and N terms are known. Conversely, when their $\overline{D}/N$ ratio has been established, an equation similar to Equation 14 can be used to compute the M_s value of the magnet using M_e and M_h measurements made in the field, $H_a = H^A/2$.

In conclusion, it is postulated that the intrinsic coercive force for the Stoner-Wohlfarth spherical model is

$$H_{ci} = 0.48[H^A - M_s(D_a + D_b)/2] = 0.48[H^A - 2\pi M_s], \quad (15)$$

when demagnetization factors are included in the computations. While direct experimental proof is lacking to support this conclusion, ample evidence has been provided to show that $\bar{D} = 2\pi$ is the proper value to use when the field,

TABLE IV Comparing the experimental values of $4\pi M_s$ and $H_i{}^A$ with the corresponding single-crystal terms

Type magnet	$4\pi M_s$ (G) calc.	$4\pi M_s$ (G) published	$H_i{}^A$ (kOe) obs.	H^A (kOe) published
$BaFe_{12}O_{19}{}^{I}$	4765	4775	17.3	17.3
$BaFe_{12}O_{19}{}^{III}$	4800	4775	17.3	17.3
$BaFe_{12}O_{19}{}^{IV}$	4780	4775	17.3	17.3
$SrFe_{12}O_{19}{}^{II}$	4770	4775	18.7	18.8
$SrFe_{12}O_{19}{}^{V}$	4770	4775	18.8	18.8
$SrFe_{12}O_{19}{}^{VI}$	4800	4775	18.7	18.8
$SrFe_{12}O_{19}{}^{VII}$	4790	4775	18.7	18.8

Note: Superscripts refer to footnotes following Table I.

$H_a = H^A/2$ is applied to the magnet. There is no reason to believe that this value of $\overline{D}$ should change when the field, $H_a = H^A/2$ is reversed in direction.

Appendix 1

The demagnetization factors for a flat plate, infinite in extent, are [3] $D_b = D_c = 0$ and $D_a = 4\pi$ in terms used by Stoner and Wohlfarth [12]. Precedent [1] is followed by assuming that these factors apply to single-domain ferrite particles. Then, in order to derive an expression for the free energy of demagnetization for these particles in terms involving the rotation of $D_a\mathbf{M}_s$ from the c-axis, the following equation [17] is invoked:

$$c = \left(\frac{\partial^2 E_D}{\partial\theta^2}\right)_{\theta=0} . \tag{16}$$

In this expression, c is a constant and θ is the angle of rotation of $D_a\mathbf{M}_s$ from the c-axes.

By means of this expression and the relationships [3], $\mathbf{H}_D = -D_a\mathbf{M}$, $dE_D = -\mathbf{H}_D d\mathbf{M}$, and $E_D = D_a M^2/2$, it is found that

$$c = \frac{2}{M_s^2}\left(\frac{\partial^2 E_D}{\partial\theta^2}\right)_{\theta=0} . \tag{17}$$

Here, H_D is the demagnetization field resulting from shape anisotropy. Furthermore, by means of these relationships, it is found that

$$-D_a = \left(\frac{d^2 E_D}{d\mathbf{M}^2}\right) . \tag{18}$$

The term, $-D_a = -4\pi$, in Equation 18 is equated to c and the variation of E_D with θ is expressed as a cosine function, thereby, deriving the equation,

$$\frac{2}{M_s^2}\left(\frac{\partial^2 E_D}{\partial\theta^2}\right) = -4\pi\cos 2\theta . \tag{19}$$

The term, $\cos 2\theta$, is used here to satisfy both the relationships on which this equation is based and the symmetry conditions imposed by uniaxial crystals. These conditions demand that only even powers of $\cos\theta$ appear in expressions for the free energy of anisotropy [2].

On integrating Equation 19, it is found that

$$E_D = \pi M_s^2\cos^2\theta + c_1 \tag{19}$$

per unit volume. Here, c_1 is an integration constant.

The free energy expressed by this equation and by Equation 2 are added, thereby, obtaining

$$E = K_1\sin^2\theta + K_2\sin^4\theta + \pi M_s^2\cos^2\theta + c_1 - HM_s\cos(\alpha - \theta) . \tag{20}$$

Here, E is the free energy of magnetization by the rotational process when energy terms resulting from strain and exchange forces are neglected.

Differentiating E with respect to θ and equating $(\partial E/\partial\theta)_\alpha$ to 0 yields the equation,

$$(2K_1/M_s - 2\pi M_s)\sin 2\theta = 2H\sin(\alpha - \theta) , \tag{21}$$

provided $K_1 \gg K_2$. This equation obviously supports the value advanced, $\overline{D} = 2\pi$, for an isotropic, spherical specimen and shows that $H_{eff}^A = H^A - 2\pi M_s$. Furthermore, it suggests that

$$H_{ci} = 0.48[H^A - 2\pi M_s] , \tag{22}$$

on the condition that replacing H^A with H_{eff}^A does not effect the value of H_{ci}.

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